Testing a theory?
Useful to contrast its predictions with alternative theories

Example: “PPN” formalism to study weak-field gravity (order Newton \( \times \frac{1}{c^2} \))

[Edington, Schiff, Baierlin, Nordtvedt, Will]

\[
\begin{align*}
- g_{00} &= 1 - 2 \frac{Gm}{rc^2} + 2 \beta_{\text{PPN}} \left( \frac{Gm}{rc^2} \right)^2 + \ldots \\
g_{ij} &= \delta_{ij} \left[ 1 + 2 \gamma_{\text{PPN}} \frac{Gm}{rc^2} + \ldots \right]
\end{align*}
\]
Solar-system experiments in the **Parametrized Post-Newtonian** formalism

**General Relativity** is essentially the only theory consistent with weak-field experiments.
GENERAL RELATIVITY is essentially the only theory consistent with weak-field experiments.
Weak-field experiments

\[
\begin{align*}
-g_{00} &= 1 - 2 \frac{G m}{r c^2} + 2 \beta_{\text{PN}} \left( \frac{G m}{r c^2} \right)^2 + \ldots \\
g_{ij} &= \delta_{ij} \left[ 1 + 2 \gamma_{\text{PN}} \frac{G m}{r c^2} + \ldots \right]
\end{align*}
\]

Strong-field tests?

- solar system
- neutron star
- black hole
- self-gravity \( \frac{G m}{R c^2} \)
- deviation from flat space

\begin{align*}
\text{binary pulsars} & \\
\text{moving clock} & \\
giving information & \\
about this strong-gravity region
\end{align*}
Many NS-WD binaries

<table>
<thead>
<tr>
<th>Pulsar</th>
<th>Companion or Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSR J1141–6545</td>
<td>2.1 m⊙ NS!</td>
</tr>
<tr>
<td>PSR J0407+1607</td>
<td></td>
</tr>
<tr>
<td>PSR J2016+1947</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>PSR J0751+1807</td>
<td></td>
</tr>
</tbody>
</table>

8 NS-NS binaries

<table>
<thead>
<tr>
<th>Pulsar</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSR B1913+16</td>
<td>[Hulse-Taylor 1974]</td>
</tr>
<tr>
<td>PSR B1534+12</td>
<td>[Wolszczan 1991]</td>
</tr>
<tr>
<td>PSR J0737–3039</td>
<td>[Burgay et al. 2003]</td>
</tr>
<tr>
<td>PSR B2127+11C</td>
<td>(in globular cluster)</td>
</tr>
<tr>
<td>PSR J1756–2251</td>
<td>[Faulkner et al. 2004]</td>
</tr>
<tr>
<td>PSR J1518+4904</td>
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<tr>
<td>PSR J1829+2456</td>
<td></td>
</tr>
<tr>
<td>PSR J1906+07</td>
<td>[Lorimer et al. 2005]</td>
</tr>
<tr>
<td></td>
<td>(maybe NS-WD?)</td>
</tr>
</tbody>
</table>
• ~ 1600 known pulsars
• ~ 100 binary pulsars

Many NS-WD binaries

PSR J1141–6545 \[\text{[Kaspi et al. 1999]}\]

Precision tests of
strong-field gravity

PSR J0407+1607
PSR J2016+1947
...

PSR J0751+1807 \[\text{[Nice et al. 2005]}\]

⇒ 2.1 m⊙ NS!

8 NS-NS binaries

\{ 
PSR B1913+16 \[\text{[Hulse-Taylor 1974]}\] 
PSR B1534+12 \[\text{[Wolszczan 1991]}\] 
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\}

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**Many NS-WD binaries**

- PSR J1141–6545
  - [Kaspi et al. 1999]

**8 NS-NS binaries**

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  - [Hulse-Taylor 1974]
- PSR B1534+12
  - [Wolszczan 1991]
- PSR J0737–3039
  - [Burgay et al. 2003]

**Small-e binaries**

- PSR J0407+1607
- PSR J2016+1947
- ...  

**null tests of GR’s symmetries**

- PSR J0751+1807
  - [Nice et al. 2005]

- 2.1 m⊙ NS!

**Precision tests of strong-field gravity**

- PSR B2127+11C (in globular cluster)

- PSR J1756–2251
  - [Faulkner et al. 2004]

- PSR J1518+4904

- PSR J1811–1736

- PSR J1829+2456

- PSR J1906+07
  - [Lorimer et al. 2005]  
  - (maybe NS-WD?)
Tests of the “strong equivalence principle” and of preferred-frame effects

• The different accelerations (due to a third body or to their absolute velocity with respect to a preferred frame) induce a polarization of the periastron towards a precise direction

\[ |e_{\text{Fixed}}| \propto |\vec{a}_A - \vec{a}_B| \]

• \( \exists \) several binary pulsars with \( \vec{e} \approx 0 \)

\[ \Rightarrow \text{statistical argument to constrain PPN parameters} \]

[Damour, Schäfer, GEF, Bell, Camilo, Wex, Stairs, …]
Tests of the “strong equivalence principle” and of preferred-frame effects

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\[ \Rightarrow \text{statistical argument to constrain PPN parameters} \]

\[ [\text{Damour, Schäfer, GEF, Bell, Camilo, Wex, Stairs, …}] \]

\[ \Rightarrow \text{Constraints on PPN parameters} \]

- \( |\alpha_1| < 1.4 \times 10^{-4} \) (\( \approx \) solar system bounds)
- \( |\alpha_3| < 4 \times 10^{-20} \) (10^{12} tighter than sol. syst.!)}
- \( |1 - m_g/m_i| < 5.6 \times 10^{-3} \) for a neutron star

[Stairs et al. 2005]
Binary-pulsar tests

- pulsar = (very stable) clock
- binary pulsar = moving clock

- Time of flight across orbit \( \propto \frac{\text{size of orbit}}{c} \) ("Roemer time delay")
  - orbital period \( P \)
  - eccentricity \( e \)
  - periastron angular position \( \omega \)
  - …

- Redshift \( \propto \frac{G m_B}{r_{AB} c^2} \) + second order Doppler effect \( \propto \frac{\sqrt{v_A^2}}{2 c^2} \) ("Einstein time delay")
  - parameter \( \gamma_{\text{Timing}} \)

- Time evolution of Keplerian parameters
  - periastron advance \( \dot{\omega} \) (order \( \frac{1}{c^2} \))
  - gravitational radiation damping \( \dot{P} \) (order \( \frac{1}{c^3} \))

"post-Keplerian" observables

[PSR B1913+16 • Hulse & Taylor]

\[
\begin{align*}
3 - 2 &= 1 \\
\text{observables} &\quad \text{unknown masses } m_A, m_B
\end{align*}
\]

Plot the three curves [strips]

\[
\begin{align*}
\gamma_{\text{Timing}}(m_A, m_B) &= \gamma_{\text{Timing}}^{\text{observed}} \\
\dot{\omega}(m_A, m_B) &= \dot{\omega}^{\text{observed}} \\
\dot{P}(m_A, m_B) &= \dot{P}^{\text{observed}}
\end{align*}
\]

" \( \gamma - \dot{\omega} - \dot{P} \) test "

Discovered by R. Hulse and J. Taylor in 1974
PSR B1534+12 in general relativity

Discovered by A. Wolszczan in 1991

5 observables $\Box$ 2 masses = 3 tests

“Galactic” contribution to $\dot{P}$

$\dot{P} \propto n/v \frac{d \text{Doppler}}{dt} \propto n/a + \frac{v^2}{d_{\text{PSR}}}$
Asymmetrical system neutron star—white dwarf

Neutron star born after white dwarf
\( \Rightarrow \) eccentricity \( e = 0.17 \) large
and nonrecycled pulsar

\[ \dot{P} = -4 \times 10^{-13} \]

Mass function

\[
\frac{(m_B \sin i)^3}{(m_A + m_B)^2} = \left(\frac{2\pi}{P}\right)^2 \frac{(x c)^3}{G}
\]
PSR J0737–3039 in general relativity

Timing Burgay et al. 2003, Double pulsar Lyne et al. 2004

\[ P = 2 \text{ h} 27 \text{ min} 14.5350 \text{ s} \]

\[ \dot{\omega} = 16.90^\circ/\text{yr} \]

\[ \frac{x_B}{x_A} = \frac{m_A}{m_B} = 1.07 \]

6 observables – 2 masses = 4 tests
The most natural theories of gravity include a scalar field besides the metric $g_{\mu\nu}$.

Tensor–scalar theories

$$S = \frac{1}{16\pi G} \sqrt{-g} \left\{ R - 2 \left( \partial \Phi \right)^2 \right\} + S_{\text{matter}} \left[ g_{\mu\nu} = e^{2\alpha(\Phi)} g_{\mu\nu} \right]$$

$$a(\Phi) = \alpha_0 (\Phi - \Phi_0) + \frac{1}{2} \alpha_0 (\Phi - \Phi_0)^2 + \ldots$$

$$\begin{align*}
\{ G_{\text{eff}} &= G \left( 1 + \frac{\Phi_0^2}{\alpha_0^2} \right) \\
\square_{\text{PPN}} - 1 &\propto \Phi_0^2
\end{align*}$$

Vertical axis ($\Phi_0 = 0$): Jordan–Fierz–Brans–Dicke theory $\Phi_0^2 = \frac{1}{2} \Phi_{\text{BD}}^2 + 3$.
Deviations from general relativity due to the scalar field

• At any order in \( \frac{1}{c^n} \), the deviations involve at least two \( \Box_0 \) factors:

\[
\text{deviations} = \Box_0^2 \left[ a_0 + a_1 \frac{Gm}{Rc^2} + a_2 \left( \frac{Gm}{Rc^2} \right)^2 + \ldots \right] < 10^{-5}
\]

LARGE for \( \frac{Gm}{Rc^2} \geq 0.2 \)

= small deviations!

• But nonperturbative strong-field effects can occur:
PSR B1913+16 in scalar-tensor theories

A tensor–scalar theory which passes the test
($\beta_0 = -4.5, \alpha_0$ small enough)

A tensor–scalar theory which does not pass the test
($\beta_0 = -6, \text{any } \alpha_0$)
Solar-system & PSR B1913+16 constraints on scalar-tensor theories of gravity

Vertical axis ($\beta_0 = 0$): Jordan–Fierz–Brans–Dicke theory

Horizontal axis ($\alpha_0 = 0$): perturbatively equivalent to G.R.

Binary pulsars impose $\beta_0 > -4.5$

i.e. $\frac{\beta_{\text{PPN}}}{\gamma_{\text{PPN}}} < 1.1$

$\alpha_0^2 = \frac{1}{2} \omega_{\text{BD}} + 3$

[T. Damour & G.E-F 1998]
Solar-system & PSR B1913+16 constraints on scalar-tensor theories of gravity

Vertical axis ($\beta_0 = 0$): Jordan–Fierz–Brans–Dicke theory

Horizontal axis ($\alpha_0 = 0$): perturbatively equivalent to G.R.

Binary pulsars impose $\beta_0 > -4.5$

\[ \frac{\Gamma_{\text{PPN}}}{\lambda_{\text{PPN}}} < 1.1 \]

Vertical axis ($\beta_0 = 0$) and Horizontal axis ($\alpha_0 = 0$) relations:

\[ \alpha_0^2 = \frac{1}{2} \omega_{\text{BD}} + 3 \]
The four accurately timed binary pulsars in general relativity

- PSR B1913+16
- PSR J1141-6545
- PSR J0737-3039
- PSR B1534+12
Solar-system & best binary-pulsar constraints on scalar-tensor theories of gravity

Vertical axis ($b_0 = 0$): Jordan–Fierz–Brans–Dicke theory

Horizontal axis ($a_0 = 0$): perturbatively equivalent to G.R.

[$T. \text{Damour} \& G.E.F \ 2006$]

$$a_0 < 0 \quad b_0 > 0$$

$$a_0 > 0$$

$$|a_0|$$

$$j_{\text{matter}}$$

$$b_0$$

$$j_{\text{binary pulsars}}$$

$$|b_0| > 4.5$$

$$\frac{\text{PPN}}{\text{PPN}} - 1 < 1.1$$

$$\frac{1}{2} \Box_{\text{BD}} + 3$$
All solar-system & binary-pulsar constraints on tensor–scalar theories

![Graph showing matter-scalar coupling function and constraints on tensor–scalar theories.]

- **LIGO/VIRGO NS-BH**
- **LIGO/VIRGO NS-NS**
- **SEP**
- **All pulsars**
- **General relativity**

- Key pulsars: B1534+12, B1913+16, J0737–3039, J1141–6545

The graph illustrates the constraints on tensor–scalar theories using matter-scalar coupling function parameters $a(\jmath)$, $b(\jmath)$, and $c(\jmath)$, with specific constraints for different astrophysical observations.
All solar-system & binary-pulsar constraints on tensor–scalar theories

\[ \alpha_j \]

\[ a_{0} < 0 \]

\[ b_{0} > 0 \]

matter-scalar coupling function

\[ a(\phi) \]

\[ \beta_0 < 0 \]

\[ \beta_0 > 0 \]

\[ |a_0| \]

LIGO/VIRGO

NS-BH

B1534+12

SEP

J0737–3039

B1913+16

All pulsars

J1141–6545

LLR

Cassini

Logarithmic scale for \( \alpha_0 \)

general relativity

(\( \alpha_0 = \beta_0 = 0 \))

LIGO/VIRGO

NS-NS
All solar-system & binary-pulsar constraints on tensor–scalar theories

\[ a(j) \]

\[ b_0 < 0 \]
\[ b_0 > 0 \]
\[ a_0 \]

matter–scalar coupling function

\[ B_{1913+16} \]
\[ J_{1141–6545} \]

Logarithmic scale for \( \alpha_0 \)

general relativity
\( (\alpha_0 = \beta_0 = 0) \)
All solar-system & binary-pulsar constraints on tensor–scalar theories

matter-scalar coupling function $a(\varphi)$

- $\alpha_0 < 0$
- $\beta_0 > 0$
- $a_0 < 0$
- $b_0 > 0$

$|a_0|$ vs $\beta_0$

- All pulsars
- Cassini
- General relativity ($\alpha_0 = \beta_0 = 0$)
All solar-system & binary-pulsar constraints on tensor–scalar theories

matter-scalar coupling function $a(\varphi)

$\beta_0 < 0$

$\beta_0 > 0$

$|a_0|$

All pulsars

LIGO/VIRGO

NS-BH

Cassini

general relativity ($\alpha_0 = \beta_0 = 0$)
All solar-system & binary-pulsar constraints on tensor–scalar theories

\[ a_j \]

\[ b_0 > 0 \]

matter-scalar coupling function

\[ a(\phi) \]

\[ \beta_0 < 0 \]

\[ \beta_0 > 0 \]

\[ |a_0| \]

\[ 10^{-2} \]

\[ 10^{-3} \]

\[ 10^{-4} \]

All pulsars

LISA NS-BH

Cassini

J1141–6545 + 1% Pdot

general relativity

\[ (a_0 = \beta_0 = 0) \]
All solar-system & binary-pulsar constraints on tensor–scalar theories

matter-scalar coupling function

$\beta_0 < 0$

$\beta_0 > 0$

$\alpha_0$

$|\alpha_0|$

$|\beta_0|$

general relativity

$(\alpha_0 = \beta_0 = 0)$

PSR-BH

LISA

NS-BH

Cassini

J1141–6545 + 1% Pdot

All pulsars
**Conclusions**

- **Binary pulsars** are ideal tools for testing the *strong-field* regime of gravity (qualitatively different from solar-system tests).
- \[ \Rightarrow \text{GR wave templates suffice for LIGO/VIRGO/LISA.} \]
- **General relativity** passes all the tests with flying colors.
- Double pulsar **PSR J0737–3039** fantastic system to test GR itself and the physics of neutron stars.
- **Best systems** for constraining scalar-tensor theories (large *dipolar* scalar waves):
  - asymmetrical Neutron Star–White Dwarf or Neutron Star–Black Hole
- Best known system: **PSR J1141–6545**